Introduction to Electrical Engineering 2
Assignment Solution 1

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1. Question 1 Solution

1.1 Section 1

Following equations describe current-voltage relations between some[1] circuit elements:

$$v_R = iR \tag{1.1}$$

$$i_C = C\dot{v}_C \tag{1.2}$$

$$v_L = L\dot{i}_L \tag{1.3}$$

Applying nodal method at the v_L node of circuit on Figure 1

$$I_0 = \frac{v_L}{R} + i_L \tag{1.4}$$

Using (1.3) one could obtain a differential equation

$$\boxed{\dot{i_L}(t) + \tfrac{R}{L} i_L(t) = \tfrac{R}{L} I_0 \quad , t > 0} \tag{1.5}$$

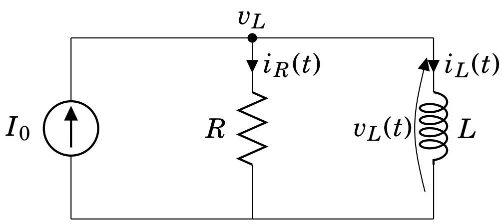


Figure 1: Electric Circuit

1.2 Section 2

Assuming circuit's steady state, yields constant[2] current, therefore

$$\textit{ZSR:} \quad \boxed{i_L = I_0} \tag{1.6}$$

Proof. Solving ZIR of the equation (1.5) :

$$i_{L,ZIR}(t) = A\exp(\frac{-t}{\tau}) \tag{1.7}$$

$$\lim_{t\to\infty} i_L(t) = \lim_{t\to\infty} \left(i_{L,ZIR} + i_{L,ZSR}\right) = i_{L,ZSR} = I_0$$

□

[1] Resistor, Capacitor and Inductor

[2] Resistor Shortened by Inductor

1.3 Section 3

Summarizing equation (1.6) with (1.7) and activating given initial condition $i_L(t=0) = 2$ A results in final solution $i_L(t)$, where $\tau = \frac{L}{R}$:

$$i_L(t) = I_0 + (2 - I_0)exp\left(\frac{-t}{\tau}\right) \quad ,t > 0$$

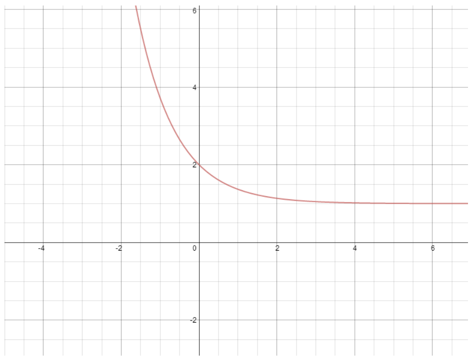


Figure 2: Inductor Current with $I_0 = 1A$

2. Question 2 Solution

2.1 Section 1

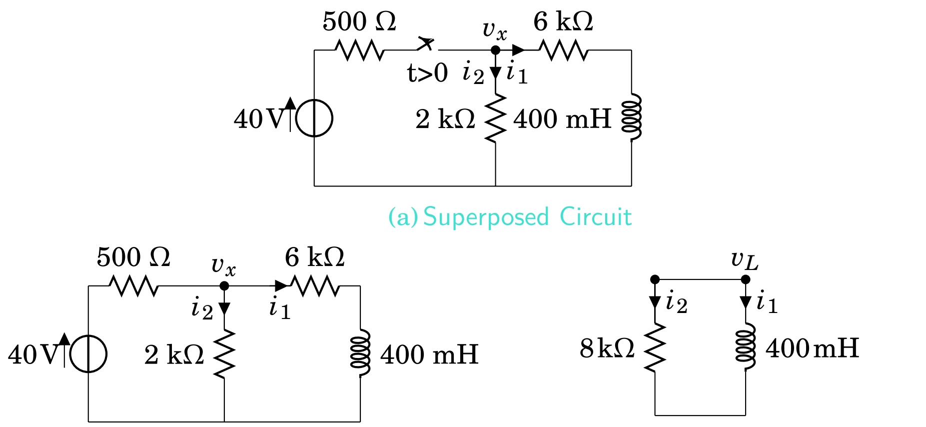
Consider the circuit on Figure 3b. Now[3] applying current divider technique one could easily calculate the following currents:

$$i_1 = \frac{40}{500 + 2k||6k} \cdot \frac{2k}{500 + 2k||6k}$$

$$i_2 = \frac{40}{500 + 2k||6k} \cdot \frac{6k}{500 + 2k||6k}$$

$$i_1(0^-) = 20\,\text{mA} \qquad i_2(0^-) = 60\,\text{mA}$$

[3] Inductor is Short



(a) Superposed Circuit

(b) Past Time Circuit

(c) Future Time Circuit

Figure 3: Question 2 Circuit

2.2 Section 2

Assuming continuity in the inductor current $i_1(0^-) = i_1(0^+) = 20\,\text{mA}$ on Figure 3c, where $i_1 = -i_2$, thus, setting $i_1(0^+) = -i_2(0^+) = 20\,\text{mA}$ we get the currents

$$i_1(0^+) = 20\,\text{mA} \qquad i_2(0^+) = -20\,\text{mA}$$

2.3 Section 3

It's obvious, that circuits 3b and 3c are equivalent to the one on Figure 1 (Page 1) with $\tau = 50$ ms and $I_0 = V_0/R_{eq} = 20$ mA:

$$\dot{i_L}(t) + \frac{R}{L} i_L(t) = \frac{R}{L} I_0 \quad , t > 0 \tag{2.8}$$

2.4 Section 4

Adopting ZIR[4] solution (1.7)

$$i_1(t) = i_1(0^+) \exp\left(\frac{-t}{\tau}\right) \quad , t > 0$$

$$i_2(t) = -i_1(t)$$

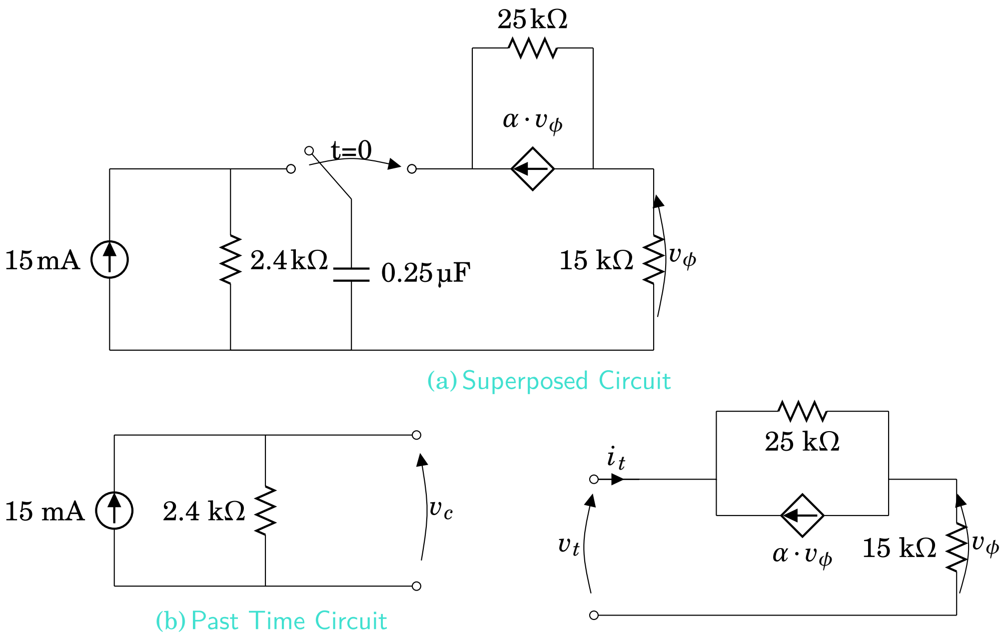
[4] Since, there is no source in the circuit

3. Question 3 Solution

3.1 Section 1

The voltage drop on the capacitor[5] on Figure 4b as follows

$$v_C(0^-) = 15\text{ mA} \cdot 2.4\text{ k}\Omega = 36\text{ V} \tag{3.9}$$



(a) Superposed Circuit

(b) Past Time Circuit

(c) Future Time Circuit

Figure 4: Question 3 Circuit

3.2 Section 2

Replacing capacitor with a test voltage, as shown on Figure 4c, allows input resistance calculation $R_{in} = \frac{v_t}{i_t}$

$$v_t - v_\phi = 25k\alpha \cdot v_\phi$$
$$v_t = (1 + 25k\alpha) \cdot v_\phi = 15k(1 + 25k\alpha) \cdot i_t$$
$$R_{in}(C) = 15k(1 + 25k\alpha) \tag{3.10}$$

[5] Open-Circuited Capacitor

3.3 Section 3

Given circuit's time constant $\tau = R \cdot C = 25$ ms requires resistor $R = 100$ kΩ. Setting into (3.10) leads to the value of α-parameter

$$\alpha \approx 2.26 \cdot 10^{-4}$$

3.4 Section 4

As before[6] using ZIR solution (1.7) adapted for voltages, where $\tau = 25$ ms and initial condition of the capacitor[7] $v_C(0^-) = v_C(0^+) = 36$ V

$$\begin{cases} v_C(t) = v_C(0^+)\exp\left(\dfrac{-t}{\tau}\right) & ,t \geqslant 0 \\ v_C(t) = v_C(0^-) & ,t < 0 \end{cases}$$

[6] Due to circuits analogy

[7] Continuity in voltage drop