

Often referred to as the most beautiful equation in mathematics, Euler's Identity,

$$e^{i\pi} + 1 = 0$$

involves the five most important constants; the additive identity 0, the multiplicative identity 1, the imaginary number i , and the two irrational numbers e and π . It should feel crazy that this is true! The goal of this project is to see **why** this equation works and use infinite series in a new way.

You should present your work on this project in a written format for a reader learning about infinite series for the first time; think about yourself at the start of this chapter of material. Explain what infinite series are, convergence vs. divergence, and how series can represent functions.

You want to show that $e^{i\pi} + 1 = 0$. The main idea is to use infinite power series to show Euler's Formula

$$e^{i\theta} = \cos \theta + i \sin \theta.$$

The imaginary number i that appears here may be new to you. The only fact you really need for this is that $i^2 = -1$. We can multiply i by real numbers as well as i itself where simplifications can be made. For example:

$$(-2i)^3 = (-2)^3 i^3 = -8(i^2)(i) = -8(-1)i = 8i$$

You do not need to delve into the world of complex analysis. . . So, to check convergence in this assignment, it is safe to assume that the absolute value of i is one, and adjustments can be made accordingly if needed. So,

$$\left| -\frac{1}{2}i \right| = \left| -\frac{1}{2} \right| |i| = \frac{1}{2}$$

It is up to you to organize this work and your explanation in a logical way. Keep the intended audience in mind as well as the guidelines for written work found in the *Specifications for Calculus Work* document.