TABLE S1: Estimated critical screening parameters of Hulthén potential, for some high-lying states having $n = 6 - 10, \ell = 0 - 9$, along with literature results.

State	δ_c		State	δ_c	
	PR†	Literature		PR†	Literature
$6s$	0.0555555	0.055556^a	$6g$	0.04058464	0.040585^a,0.04058464^b
$7s$	0.0408163	0.040816^a	$7g$	0.03135273	0.031353^a,0.03135273^b
$8s$	0.0312499	0.031250^a	$8g$	0.0249258	0.024926^a,0.0249258^b
$9s$	0.0246913	0.024691^a	$9g$	0.0202774	0.020278^a,0.0202774^b
$10s$	0.0199999	0.020000^a	$10g$	0.0168095	0.016810^a,0.0168095^b
$6p$	0.0515788	0.051579^a,0.051536^b	$6h$	0.03750415	0.037504^a,0.03750415^b
$7p$	0.0383973	0.038398^a,0.038365^b	$7h$	0.02928423	0.029284^a,0.02928423^b
$8p$	0.0296803	0.029681^a,0.029654^b	$8h$	0.02347828	0.023478^a,0.02347828^b
$9p$	0.0236212	0.023621^a,0.023599^b	$9h$	0.0192297	0.019230^a,0.0192297^b
$10p$	0.0192398	0.019240^a,0.019222^b	$10h$	0.0160298	0.016030^a,0.0160298^b
$6d$	0.04766137	0.047661^a,0.0476580^b	$7i$	0.02737901	0.027379^a
$7d$	0.0359476	0.035948^a,0.0359445^b	$8i$	0.02212412	0.022124^a
$8d$	0.0280578	0.028058^a,0.0280547^b	$9i$	0.0182370	0.018237^a
$9d$	0.0224966	0.022497^a,0.0224936^b	$10i$	0.0152833	0.015283^a
$10d$	0.0184326	0.018433^a,0.0184299^b	$8k$	0.02086426	0.020864^a
$6f$	0.04397459	0.043975^a,0.04397452^b	$9k$	0.01730265	0.017303^a
$7f$	0.03358103	0.033581^a,0.03358094^b	$10k$	0.0145735	0.014573^a
$8f$	0.0264591	0.026459^a,0.02645904^b	$9l$	0.01642647	0.016427^a
$9f$	0.0213719	0.021372^a,0.02137183^b	$10l$	0.0139017	0.013902^a
$10f$	0.0176149	0.017615^a,0.0176147^b	$10m$	0.0132679	0.013268^a

[a]Ref. [1]. [b]Ref. [2]. †PR implies Present Result.

TABLE S2: Estimated critical screening parameters of Yukawa potential, for some high-lying states having $n = 6 - 10, \ell = 0 - 9$, along with literature results. PR implies Present Result.

State	δ_c		State	δ_c	
	PR	Literature [3]		PR	Literature [3]
$6s$	0.035182	0.03518	$6g$	0.023799103	0.02380
$7s$	0.025874	0.0258	$7g$	0.018646215	0.01864
$8s$	0.019824	0.0198	$8g$	0.014980862	0.01498
$9s$	0.015672	0.0156	$9g$	0.012286145	0.01228
$10s$	0.012699		$10g$	0.010250170	
$6p$	0.032174932	0.03217	$6h$	0.021524548	0.02152
$7p$	0.024047639	0.0240	$7h$	0.017095135	0.01709
$8p$	0.018640705	0.01864	$8h$	0.013883519	0.01388
$9p$	0.014865869	0.01486	$9h$	0.011485753	0.01148
$10p$	0.012128229		$10h$	0.009651169	
$6d$	0.029166650	0.02916	$7i$	0.015691083	0.01569
$7d$	0.022161826	0.02216	$8i$	0.012871464	0.01287
$8d$	0.017390648	0.01739	$9i$	0.010736147	0.01073
$9d$	0.013999880	0.01400	$10i$	0.009082952	
$10d$	0.011506513		$8k$	0.011944531	0.01194
$6f$	0.026350671	0.02635	$9k$	0.010039758	0.01003
$7f$	0.020342170	0.02034	$10k$	0.008548707	
$8f$	0.016156534	0.01615	$9l$	0.009395999	0.00939
$9f$	0.013129670	0.01313	$10l$	0.008049285	
$10f$	0.010872967		$10m$	0.007584125	

TABLE S3: Estimated critical screening parameters of ECSC potential, for some high-lying states having $n = 6 - 10, \ell = 0 - 9$, along with literature results. PR implies Present Result.

State	δ_c		State	δ_c	
	PR§	Literature		PR§	Literature
$6s$	0.01787828	0.01787828^a,0.01787790^b	$6g$	0.0160994830	0.01609948^a,0.016099483^b
$7s$	0.01312287	0.01312287^a,0.01312275^b	$7g$	0.0121108414	0.01211084^a,0.012110841^b
$8s$	0.010041420	0.01004142^a,0.01004138^b	$8g$	0.0094255746	0.00942557^a,0.009425574^b
$9s$	0.007930924		$9g$	0.0075357713	
$10s$	0.0064223221		$10g$	0.0061576653	
$6p$	0.0176520702	0.01765207^a,0.0176520692^b	$6h$	0.0154554769	0.01545548^a,0.015455476^b
$7p$	0.0130010639	0.01300107^a,0.013001062^b	$7h$	0.0117204888	0.01172049^a,0.011720488^b
$8p$	0.0099700872	0.00997009^a,0.009970085^b	$8h$	0.0091765721	0.00917657^a,0.009176572^b
$9p$	0.0078864055		$9h$	0.0073701634	
$10p$	0.0063931148		$10h$	0.0060436156	
$6d$	0.0172429036	0.01724290^a,0.017242903^b	$7i$	0.0113144150	0.01131442^a,0.011314415^b
$7d$	0.0127747014	0.01277470^a,0.012774701^b	$8i$	0.0089121305	0.00891213^a,0.008912130^b
$8d$	0.0098352041	0.00983521^a,0.009835204^b	$9i$	0.0071912774	
$9d$	0.0078012274		$10i$	0.0059186845	
$10d$	0.0063367620		$8k$	0.0086398532	0.00863985^a,0.008639853^b
$6f$	0.0167081500	0.01670815^a,0.016708150^b	$9k$	0.0070041846	
$7f$	0.0124693824	0.01246938^a,0.012469382^b	$10k$	0.0057862828	
$8f$	0.0096491922	0.00964919^a,0.009649192^b	$9l$	0.0068128353	
$9f$	0.0076818589		$10l$	0.0056491977	
$10f$	0.0062568394		$10m$	0.0055096394	

[a]Ref. [7]. [b]Ref. [8].

TABLE S4: Eigenvalues (a.u.) of $n = 3, 4$ states of confined ECSC potential for $\delta = 0.02$. Numbers in the parentheses denote reference energies quoted from [9].

State	$r_c = 0.1$	$r_c = 0.5$	$r_c = 1$	$r_c = 2$	$r_c = 5$
$3s$	4406.1416518	170.60516396	40.883123723	9.3341469004	1.0731978420
				(9.33415)	(1.07320)
$3p$	2960.4823022	114.66355228	27.493994384	6.2889991502	0.7276959975
				(6.28900)	(0.72770)
$3d$	1644.5499223	63.180184177	14.987462939	3.3475046681	0.3490909625
				(3.34750)	(0.34909)
$4s$	7857.6491849	308.21724725	75.150492179	17.836089963	2.4023028763
				(17.83609)	(2.40230)
$4p$	5918.2028888	232.44795983	56.778032985	13.530580567	1.8504011627
				(13.53058)	(1.85040)
$4d$	4115.6026320	161.37700634	39.335318864	9.3341465110	1.2596272053
				(9.33415)	(1.25963)
$4f$	2426.4155489	94.646597432	22.915824203	5.3620893411	0.6894218988
				(5.36209)	(0.68942)
	$r_c = 10$	$r_c = 20$	$r_c = 30$	$r_c = 50$	$r_c = 100$
$3s$	0.1113277900	−0.0302492345	−0.0358787689	−0.0360250925	−0.0360251051
	(0.11133)			(−0.03603)	
$3p$	0.0691008416	−0.0319140038	−0.0358733580	−0.0359675961	−0.0359676034
	(0.06910)			(−0.03597)	
$3d$	0.0128160637	−0.0342064512	−0.0358194164	−0.0358506603	−0.0358506623
	(0.01282)			(−0.03585)	
$4s$	0.4250635505	0.0363462881	−0.0054277289	−0.0124953824	−0.0125717772
	(0.42506)			(−0.01250)	
$4p$	0.3359680167	0.0277302857	−0.0066764629	−0.0124281276	−0.0124857523
	(0.33597)			(−0.01243)	
$4d$	0.2223514916	0.0141166051	−0.0086778605	−0.0122798641	−0.0123102664
	(0.22235)			(−0.01228)	
$4f$	0.1081309850	−0.0003604550	−0.0106312256	−0.0120295162	−0.0120381878
	(0.10813)			(−0.01203)	

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